Periodic Preventive Maintenance Models for Deteriorating Systems With Considering Failure Limit

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Abstract

This paper is to model periodic preventive maintenance policies over an infinite time span for the deteriorating systems with minimal repair at each failure. The concept of the improvement factor method is applied to measure the restoration of a system after each PM. An improvement factor is established as a function of the system's age and the cost of each PM. Then, two periodic PM models are developed for the cases of considering and without considering failure limit. The optimal PM interval and the optimal replacement time for the two cases can be obtained by minimizing the objective functions of the cost rate through the algorithms provided by this research. An example of using Weibull failure distribution is provided to investigate the proposed models.

1. Introduction

It has been shown that the imperfect preventive maintenance can restore the age of a deteriorating system (or machine) to younger age and reduce the system's failure rate (Pham and Wang 1996). Nakagawa (1979) presents a model to describe that the age of a system is reduced by a certain units of time after performing a preventive maintenance (PM). Canfield (1986) proposes a periodic PM policy which is assumed to slow the rate of system degradation, while the hazard rate keeps monotone increase. Chan and Shaw (1993) also study the reduction of the failure rate after performing PM.

Malik (1979) proposes the improvement factor to measure the restoration of age and failure rate of a system after performing a PM. Jayabalan and Chaudhuri (1992) also use the improvement factor method to investigate the restoration effect on the age of a system after a PM. Most of the PM models with the improvement factor in the literature assume that the improvement factor be constants. Lie and Chun (1986) consider the improvement factor as a variable, yet, some parameters are not well defined. Yang et al. (2003) also propose an improvement factor which is a function of the number of PM performed and the cost of each PM.

In the literature, many PM models proposed for deteriorating systems are typically to determine the optimum interval between PMs and the number of PMs before replacing the system by minimizing the expected average cost over a finite or infinite time span. Nakagawa (1986) considers periodic and sequential PM policies for the system with minimal repair at failure and provides the optimal policies by minimizing the expected cost rates.

In this paper, the periodic PM models in an infinite time span are proposed for the deteriorating systems with minimal repair at each failure. Two PM policies with: (1) no failure limit; (2) failure limit are considered. The improvement factor developed by Yang et al. (2003) is applied in this research to measure the restoration effect of a system after each PM. The optimal PM interval and the optimal number of PM before replacement of the proposed models are determined by minimizing the cost rate. An example of using Weibull failure distribution is given to investigate the proposed models.

2. Models and assumptions

A preventive maintenance model is developed with applying the improvement factor provided by Yang et al. (2003). The assumptions of the proposed PM models are as follows.

- The system is repairable and is deteriorating over time with increasing failure rate (IFR).
- Periodic PMs with constant interval (h) are performed over an infinite time span.
- Minimal repair is performed when failure occurs between PMs.
- The system is replaced at the end of the N^{th} interval.
- The improvement factor of each PM is a variable, which is a function of the number of PM performed and the cost of PM.
- The costs of PM, minimal repair, and replacement are assumed to be constant. The cost of PM and the cost of minimal repair are not greater than the cost of replacement.
- The times to perform PM, minimal repair, and replacement are negligible.

2.1 The improvement factor

The improvement factor applied in this paper is developed by Yang et al. (2003), which is assumed to be a function of the number of PM performed and the cost of each PM. The function of this improvement factor is shown as follows.

$$\eta_i = \left(a \frac{C_{pm}}{C_{pr}} \right)^{bi} \tag{1}$$

where η_i represents the improvement factor of the i^{th} PM, $0 < \eta_i < 1$, C_{pm} is the cost of each PM, C_{pr} is the replacement cost of a system, a and b are the adjustment parameters for the improvement factor.

2.2 The effective age and the reliability function

The effective age before and after the i^{th} PM can be obtained as follows, respectively.

$$W_i^- = W_{i-1}^+ + h = (i - \sum_{j=1}^{i-1} \eta_j) h, \quad i = 2, 3, \ldots, N \tag{2}$$

$$W_i^+ = W_i^- - \eta_i h = (i - \sum_{j=1}^{i} \eta_j) h, \quad i = 1, 2, \ldots, N \tag{3}$$

where h represents the time interval between two PMs, (N-1) represents the number of PMs, and $W_1^- = h$. The effective age between $(i\text{-}1)^{th}$ and i^{th} PM is shown as below.

$$W_i(t) = W_{i-1}^+ + t, \quad \text{where } 0 < t < h. \tag{4}$$

2.3 The cost rate function of the PM model

The cost rate function of the proposed PM model can be obtained as follows.

$$C(h,N)=\frac{(N-1)C_{pm}+C_{pr}+C_{mr}\sum_{i=0}^{N-1}\int_{ih}^{(i+1)h}\lambda_i(t)dt}{Nh}=\frac{(N-1)C_{pm}+C_{pr}+C_{mr}\sum_{i=0}^{N-1}\int_{W_i^+}^{W_{i+1}^-}\lambda_0(t)dt}{Nh} \quad (5)$$

where C_{mr} is the cost of each minimal repair, $\lambda_i(t)$ is the hazard rate function between the i^{th} and the $(i+1)^{th}$ PMs, and $\lambda_0(t)$ is the original hazard rate function. In this research, Weibull failure distribution with shape parameter β and scale parameter θ is applied as a study case.

2.4 The optimal number of PMs and the optimal time to replacing a system

2.4.1 The PM model without failure rate limit

Since $\lambda_i(t)<\lambda_{i+1}(t)$ for $0\leq t\leq h$, the algorithm provided by Nakagawa (1986) can be applied to find the optimal solution, which is shown as follows.

(1) By taking the partial derivative of h of the cost rate function and letting it equal to zero, then, the solution of h, h^*, can be obtained as below.

$$h^*=\left\{\frac{(N-1)C_{pm}+C_{pr}}{C_{mr}(\beta-1)\sum_{i=1}^{N}\left[\left((i-\sum_{j=1}^{i}\eta_{j-1})/\theta\right)^{\beta}-\left((i-1-\sum_{j=1}^{i}\eta_{j-1})/\theta\right)^{\beta}\right]}\right\}^{1/\beta} \quad (6)$$

(2) Then, the optimal value of N, N^*, can be determined so that

$$N^*=\min_N C(h^*,N),\ N=1,2,\ldots \quad (7)$$

2.4.2 The PM model with failure rate limit

Suppose that the system has to be replaced when the reliability or failure rate reaches a certain level, say R^* or λ^*, respectively. Let $W_{N_R}^-$ be the effective age of which the failure rate reaches λ^* and the replacement is in the N_R^{th} PM. Then, we can obtain

$$R(W_{N_R}^-)=R^* \text{ or } \lambda(W_{N_R}^-)=\lambda^* \quad (8)$$

Thus, the periodic interval of PM for this model (h_R) can be found as

$$h_R=\left[\theta(-\ln R^*)^{1/\beta}\right]\Big/\left[N_R-\sum_{j=1}^{N_R-1}\eta_j\right] \quad (9)$$

Then, the optimal value of N, N_R, can be determined so that

$$N_R=\min_N C(h_R,N),\ N=1,2,\ldots \quad (10)$$

3. Numerical examples

From a numerical example with the following conditions: Weibull(β=10, θ=100), C_{pm} = 10000, C_{mr} = 50000, C_{pr} = 5000000, a = 1, b = 0.001 and R^* = 0.6, we can obtain the optimal solution of N^* = 18, h^* = 60.895, $T = h^*N^*$ = 1096, and $C(h^*,N^*)$ = 5241 for the case of no reliability limit; N_R = 18, h_R = 48.769, $T = h_RN_R$ = 878, and $C(h_R, N_R)$ = 5961 for the case of having reliability limit. The effects of C_{pm} and C_{mr} as well as of parameters a and b for the proposed models are shown below.

Table 1: The effect of C_{pm} and C_{mr} for the proposed models

		C_{pm}/C_{mr} = 0.5		C_{pm}/C_{mr} = 2		C_{pm}/C_{mr} = 10		C_{pm} = 10000					
		C_{mr} = 50000						C_{pm}/C_{mr} = 2		C_{pm}/C_{mr} =0.5		C_{pm}/C_{mr} = 0.1	
Contraint		No limit	Reliab. limit	No limit	Reliab. limit	No limit	Reliab. limit	No limit	Reliab. limit	No limit	Reliab. limit	No limit	Reliab. limit
h^*	h_R	62.13	52.35	71.00	58.99	91.69	75.46	76.66	48.77	66.74	48.77	56.82	48.77
N^*	N_R	19	18	19	18	17	15	18	18	18	18	18	18
$T=h^*N^*$	$T=N_Rh_R$	1181	942	1349	1062	1559	1132	1380	878	1201	878	1023	878
$C(h^*,N^*)$	$C(h_R,N_R)$	5130	5829	5601	6385	9267	10715	4163	5897	4782	5918	5617	6032
R_T	R^*	0.009	0.6	0.007	0.6	0.002	0.6	3.6E-21	0.6	7.8E-6	0.6	0.095	0.6

Table 2: The effect of parameters a and b for the proposed models

		a = 1		a = 10		a = 100		a = 1					
		b = 0.001						b = 0.0001		b =0.001		b = 0.1	
Contraint		No limit	Reliab. limit	No limit	Reliab. limit	No limit	Reliab. limit	No limit	Reliab. limit	No limit	Reliab. limit	No limit	Reliab. limit
h^*	h_R	60.90	48.77	58.57	49.77	57.21	49.55	56.78	50.62	60.90	48.77	58.44	43.00
N^*	N_R	18	18	23	22	35	34	54	53	18	18	3	3
$T=h^*N^*$	$T=N_Rh_R$	1096	878	1347	1095	2002	1685	3066	2683	1096	878	175	129
$C(h^*,N^*)$	$C(h_R,N_R)$	5241	5961	4306	4827	2963	3227	2004	2119	5241	5961	31817	39113
R_T	R^*	0.009	0.6	0.018	0.6	0.059	0.6	0.147	0.6	0.009	0.6	1.7E-5	0.6

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